

# Auctions

Part III: Dynamics — Episode 8

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# Paying Per Click

- ▶ Ads in Google's sponsored links are based on a cost-per-click model
  - ▶ Advertisers only pay when a user actually clicks on the ad
- ▶ The amount that advertisers are willing to pay per click is often surprisingly high
  - ▶ To occupy the most prominent spot for "calligraphy pens" costs about \$1.70 per click
  - ▶ For some queries, the cost per click can be stratospheric — \$50 or more for a query on "mortgage refinancing"!

**But how does a search engine set the prices per click for different queries? — it would be difficult to set these prices with so many keywords!**

**Required reading: “Networks, Crowds,  
and Markets,” Chapter 9.1 — 9.5**

**Let's first focus on a few simple types of auctions, and see how they promote different kinds of behaviour among bidders**

# Simple auctions

- ▶ Consider the case of a **seller** auctioning off one item to a set of buyers
- ▶ Assumption: a bidder has an **intrinsic value** for the item being auctioned
  - ▶ She is willing to purchase the item for a price up to this value, but no higher
  - ▶ Also called the bidder's **true value** for this item

# Four main types of auctions

- ▶ **Ascending-bid** auctions (**English** auctions)
  - ▶ Carried out interactively in real-time
  - ▶ The seller gradually raise the price
  - ▶ Bidders drop out until one bidder remains — the winner at this final price

# Four main types of auctions

- ▶ **Descending-bid** auctions (**Dutch** auctions)
  - ▶ Carried out interactively in real-time
  - ▶ Seller gradually lowers the price from some high initial value
    - ▶ until the first moment when some bidder accepts and pays the current price

# Four main types of auctions

- ▶ **First-price sealed-bid** auctions
  - ▶ Bidders submit simultaneously “sealed bids” to the seller
  - ▶ The highest bidder wins the object and pays the value of her bid

# Four main types of auctions

- ▶ **Second-price sealed-bid** auctions (Vickrey auctions)
  - ▶ Bidders submit simultaneous sealed bids to the sellers
  - ▶ The **highest bidder** wins the object and pays the value of the second-highest bid
  - ▶ William Vickrey, who proposed this type of auctions, were the first to analyze auctions with game theory (1961)

# When are auctions appropriate?

- ▶ Auctions are generally used by sellers in situations where they do not have a good estimate of the buyers' true values for an item, and where buyers do not know each other's values
  - ▶ If the intrinsic value of the buyer is known, there's no need for auctions
  - ▶ The seller (or the buyer) simply commit to a fixed price that is just below the intrinsic value of the buyer (or just above that of the seller)

# The goal of auctions

- ▶ The goal of auctions is to elicit bids from buyers that reveal these values
- ▶ Assuming that the buyers have **independent, private, true** values for the item

# Descending-Bid and First-Price Auctions

- ▶ In a descending-bid auction —
  - ▶ As the seller lowers the price from its high initial starting point, no bidder says anything until finally someone actually accepts the bid and pays the current price
  - ▶ Bidders learn nothing while the auction is running, other than the fact that no one has accepted the current price yet
  - ▶ For each bidder  $i$ , there's a first price  $b_i$  at which she would be willing to break the silence and accept the item at price  $b_i$
- ▶ It is equivalent to a **sealed-bid first-price auction**: this price  $b_i$  plays the role of bidder  $i$ 's bid
  - ▶ The item goes to the bidder with the highest bid value, and this bidder pays the value of her bid in exchange for the item

# Ascending-Bid and Second-Price Auctions

- ▶ In an ascending-bid auction —
  - ▶ Bidders gradually drop out as the seller steadily raises the price
  - ▶ The winner of the auction is the last bidder remaining, and she pays the price at which the second-to-last bidder drops out
  - ▶ For a bidder, it doesn't make sense to stay after the price exceeds her true (intrinsic and private) value
  - ▶ Or to leave before the current price reaches her true value
- ▶ A bidder stays in an ascending-bid auction up to the exact moment when the current price reaches her true value
  - ▶ The item goes to the highest bidder at a price equal to the second-highest bid
  - ▶ This is precisely the rule used in **sealed-bid second-price** auctions

# Second-Price Auctions

- ▶ **Main result:** With independent, private values, bidding your true value is a **dominant strategy** in a second-price sealed-bid auction
  - ▶ That is, the best choice of bid is exactly what the object is worth to you
- ▶ To show this, we need to formulate the second-price auction as a **game**
  - ▶ Bidders correspond to **players**
  - ▶ Let  $v_i$  be bidder  $i$ 's true value for the object
  - ▶ Bidder  $i$ 's strategy is an amount  $b_i$  to bid as a function of her true value  $v_i$

# Truthful bidding in second-price auctions

- ▶ The payoff to bidder  $i$  with value  $v_i$  and bid  $b_i$  is defined as follows:

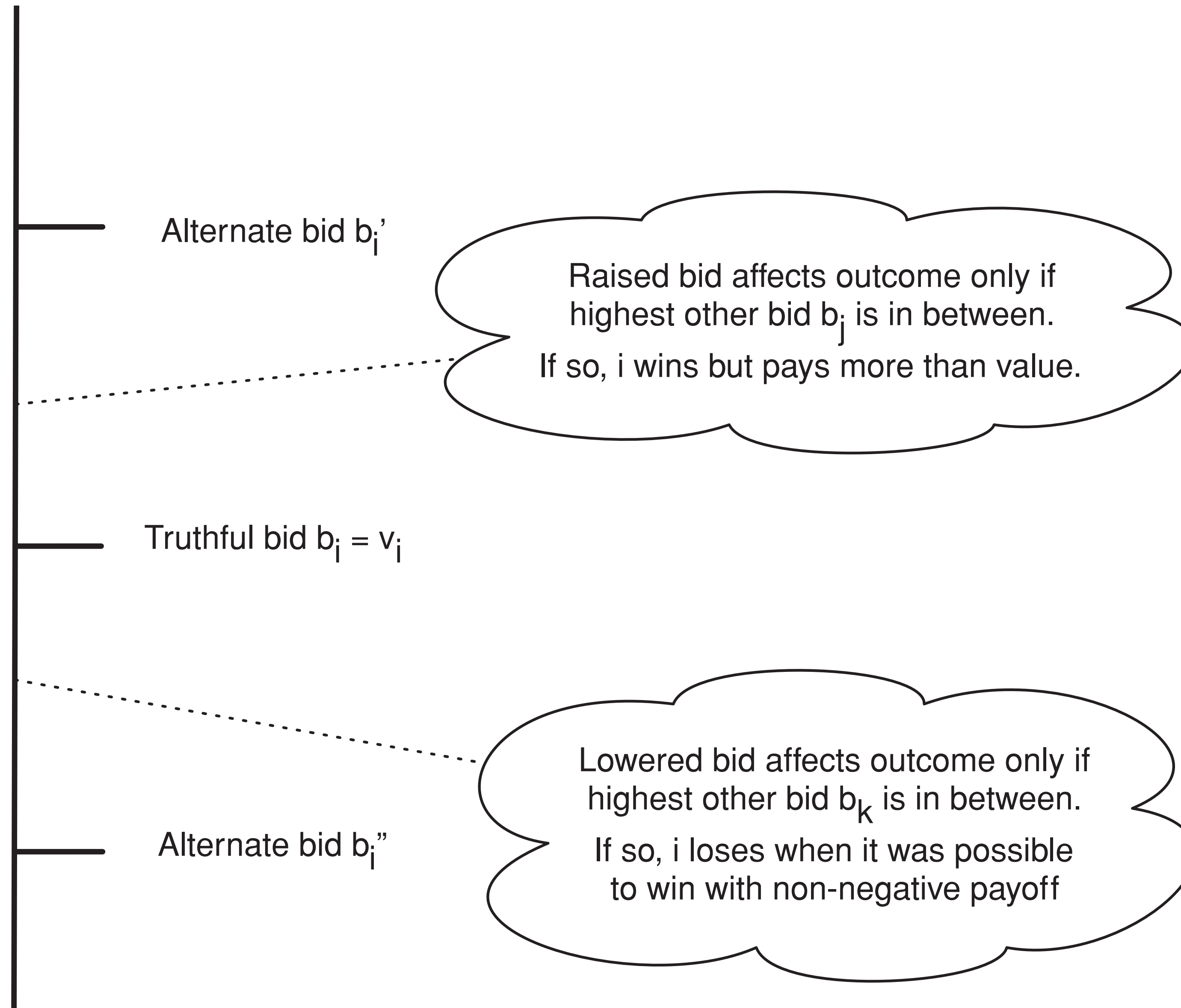
If  $b_i$  is not the winning bid, then the payoff to  $i$  is 0. If  $b_i$  is the winning bid, and some other  $b_j$  is the second-place bid, then the payoff to  $i$  is  $v_i - b_j$ .

- ▶ Claim: In a sealed-bid second-price auction, it is a **dominant strategy** for each bidder  $i$  to choose a bid  $b_i = v_i$ .

# Proving the claim

- ▶ We need to show that if bidder  $i$  bids  $b_i = v_i$ , then no deviation from this bid would improve her payoff, regardless of which strategy everyone else is using
- ▶ Two cases to consider: deviations in which  $i$  raises her bid, and deviations in which  $i$  lowers her bid
- ▶ In both cases, the value of  $i$ 's bid only affects whether  $i$  wins or loses, but it never affects how much  $i$  pays in the event that she wins
  - ▶ which is determined entirely by the other bids

# Deviating by raising or lowering her bid



# First-price auctions

- ▶ The payoff to bidder  $i$  with value  $v_i$  and bid  $b_i$  is defined as follows:

If  $b_i$  is not the winning bid, then the payoff to  $i$  is 0. If  $b_i$  is the winning bid, then the payoff to  $i$  is  $v_i - b_i$ .

- ▶ Bidding your true value is no longer a dominant strategy!
  - ▶ A payoff of 0 if you lose (as usual), and a payoff of 0 if you win, too
- ▶ The optimal way to bid is to “shade” your bid slightly downward, in order to get a positive payoff if you win
  - ▶ If it’s too close to the true value, your payoff won’t be large if you win
  - ▶ If it’s too far below, you reduce the chance of winning

**Required reading: “Networks, Crowds,  
and Markets,” Chapter 9.1—9.5**